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An estimate of the $K^0 - \overline{K^0}$ mixing amplitude

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Abstract: We computed the B_K parameter on the lattice by using a non-perturbatively improved Wilson action. From our quenched simulation we obtain $B_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.73(7)^{+0.05}_{-0.01}$. To gain some insight in the systematic errors due to the use of the quenched approximation, we also made the unquenched computation with Wilson fermions. We do not observe any significant deviation with respect to the quenched result.

Keywords: Kaon Physics, Lattice QCD.

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Contents

1. Preliminaries: motivation and status

The experimental value of the parameter ε_K, which is the measure of indirect CP violation in the kaon system, has been accurately determined [1]. After combining $|\varepsilon_K^{(\mathrm{exp.})}|$ with its theoretical expression derived in the Standard Model (SM), one obtains a hyperbola in the ($\bar\rho$, $\bar\eta$) plane [2, 3],

$$\bar\eta A^2 \hat B_K \Big(1.11(5)\cdot A^2(1-\bar\rho) \;+\; 0.31(5)\Big) = 0.226\,, \tag{1.1}$$

which, together with two equations involving B-mesons, represents one of the main constraints onto the shape of the CKM unitarity triangle. In eq. (1.1), the quantities $\lambda = 0.22$, A, $\bar\eta$, $\bar\rho$ are the standard Wolfenstein parameters of the CKM matrix elements [4, 5], while $\hat B_K$ is the so -called bag parameter which encodes the low energy QCD contribution to the $K^0-\overline{K^0}$ mixing amplitude. The main uncertainty in eq. (1.1) comes precisely from $\hat B_K$ (to which we focus in this letter) and to less extent to the parameter A which is known from $|V_{cb}| = A\lambda^2$.

We first remind the reader the definition of $\hat B_K$. As introduced in ref. [6], the parameter $\hat B_K$ measures a mismatch between the hadronic matrix element responsible for the $K^0-\overline{K^0}$ mixing and its value obtained by applying the vacuum saturation approximation (VSA), i.e.

$$\langle \bar K^0|Q(\mu)|K^0\rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)\,, \tag{1.2}$$

where μ indicates the scale dependence of the operator $Q = \bar s\gamma_\mu(1-\gamma_5)d\,\bar s\gamma_\mu(1-\gamma_5)d$, in some renormalization scheme. The QCD anomalous dimension of this operator, in the RI-MOM and $\overline{\rm MS}$ schemes, is known from perturbation theory to NLO accuracy [7]–[10]. To that precision, $B_K(\mu)$ can be converted to the renormalization group invariant (RGI) form,

$$\hat{B}_K = \alpha_s(\mu)^{-\gamma_0/2\beta_0}\left(1+\frac{\alpha_s(\mu)}{4\pi}J\right)B_K(\mu)\,, \tag{1.3}$$

which is the one that is needed in eq. (1.1). Besides the scheme independent $\gamma_0 = 4$ and $\beta_0 = 11 - 2n_f/3$, (n_f is the number of active flavours), the NLO coefficient J is renormalization scheme dependent and in the two schemes which we use in this letter, it is given by [7]–[10]

$$\begin{aligned} J_{\text{RI-MOM}} &= 8\log 2 - \frac{17397 - 2070 n_f + 104 n_f^2}{6(33-2n_f)^2}\,, \\ J_{\overline{\rm MS}} &= \frac{13095 - 1626\,n_f + 8\,n_f^2}{6\,\left(33 - 2\,n_f\right)^2}\,. \end{aligned} \tag{1.4}$$

The main task is to compute $B_K(\mu)$ in some scheme, by using a non-perturbative technique, and then to convert the result to the RGI form, $\hat{B}_K$. We choose lattice QCD and the RI-MOM scheme.

Before discussing our computation, let us briefly recapitulate the actual situation concerning the value of B_K, which at the same time gives a good motivation to compute this quantity.

Historically, the three distinct approximations applied to this problem lead to different results: VSA gave $B_K = 1$ [6], PCAC and SU(3) symmetry implied $\hat{B}_K = 1/3$ [11], and the limit of the large number of colours ($N_c \to \infty$) yielded $B_K = 3/4$ [12].[1] Such a large spread of values prompted many physicists to develop various models which typically combine the large N_c expansion with chiral perturbation theory (χPT), plus some additional assumption(s). Two most recent studies, in which such a strategy has been adopted, resulted in $\hat{B}_K = 0.41(9)$ [13] and $\hat{B}_K = 0.77(5)(5)$ [14].[2] On the other hand, the chiral quark model prediction of ref. [15] favours much larger value, i.e. $\hat{B}_K = 1.1(2)$. QCD sum rules have also been used to compute $\hat{B}_K$, with the results dispersed over a large range of values.[3]

Clearly, these refinements of the approximate results did not help clarifying the issue of the correct value for $\hat{B}_K$. It should be mentioned, however, that in SM one can extract the range of allowed values for $\hat{B}_K$, by requiring the consistency with the B-physics constraints on the CKM triangle. In this way, the authors of ref. [19] found that $0.62 \leq \hat{B}_K \leq 1.48$ (with 95% C.L.), thus excluding the low SM values such as $\hat{B}_K < 0.6$ (see [19] and references therein).

[1]Note that VSA and large N_c cannot distinguish between $B_K(\mu)$ and $\hat{B}_K$.

[2]The result of ref. [13] is obtained in the chiral limit in which the authors of ref. [14] quote $\hat{B}_K^{\chi} = 0.32(6)(12)$.

[3]Three different QCD sum rule techniques lead to: $\hat{B}_K = 0.5(1)(2)$ [16], $0.84(8)$ [17], $0.39(10)$ [18].

The most direct and systematically most improvable way to compute the matrix element (1.2) in QCD is by using the lattice simulations. Over the past decade, a substantial progress in controlling various systematic uncertainties of this approach has been made (see refs. [20, 21, 22] for reviews which contain a complete list of references).

Three currently used formulations of the lattice fermion action have been employed in the computation of $\hat{B}_K$ (or, as it is customary $B_K^{\overline{\rm MS}}(2\,{\rm GeV})$) , and in the quenched approximation, they lead to the following results:

- With the staggered fermions one can compute the matrix element (1.2) by using very light quark masses. The renormalization of the operator $Q(\mu)$ has been performed perturbatively. By extrapolating to the continuum limit, JLQCD [23] obtained
$$B_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.628(42)\,, \tag{1.5}$$
and thus improved (confirmed) the previous result of ref. [24], $B_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.62(2)(2)$.

- With Wilson fermions, as compared to the staggered, one cannot work with the very light quark masses, but the renormalization of the operator Q (which is more delicate due to the mixing with the operators of the same dimension) has been done non-perturbatively. Presently, the most refined result is the one by JLQCD [25],
$$B_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.69(7)\,, \tag{1.6}$$
fully consistent with the previous $B_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.66(11)$, as obtained in ref. [26].

- Very recently, also the domain wall fermions were employed in this computation. The results of RBC and CP-PACS are:
$$\begin{aligned} B_K^{\overline{\rm MS}}(2\,{\rm GeV}) &= 0.538(8)\ [27]\,,\\ B_K^{\overline{\rm MS}}(2\,{\rm GeV}) &= 0.575(6)\ [28]\,. \end{aligned} \tag{1.7}$$

  Notice that RBC also implemented the non-perturbative renormalization in their computation [29, 30].

In summary, from the use of various lattice techniques, one concludes that the value of $B_K^{\overline{\rm MS}}(2\,{\rm GeV})$ lies in the interval $0.53 \le B_K^{\overline{\rm MS}}(2\,{\rm GeV}) \le 0.77$, or in the RGI form, $0.72 \le \hat{B}_K \le 1.05$.[4]

[4]This range of values agrees with the one presented by L. Lellouch at “Lattice 2000”, namely $0.66 \le \hat{B}_K \le 1.06$ [21].

In this letter, we contribute the ongoing discussion by presenting the results of our computation of the matrix element (1.2), for which we use the $\mathcal{O}(a)$ non-perturbatively improved Wilson action [31], and renormalize the operator $Q(\mu)$ non-perturbatively. The operator $Q(\mu)$ has not been improved, though.[5] In order to examine the systematic uncertainty arising from the use of quenched approximation, we attempted an unquenched computation with two degenerate dynamical quarks ($n_f = 2$) and for two choices of the dynamical quark masses. From the direct comparison to the quenched result, and bearing in mind the modest statistical quality of our unquenched data, we do not see any difference between the quenched and unquenched results.

Our final numbers are:

$$B_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.73(7)^{+0.05}_{-0.01}\,; \qquad \hat{B}_K = 1.01(9)^{+0.07}_{-0.01}\,; \tag{1.8}$$

thus favouring the larger values for this quantity.

2. Outline of the lattice computations

In order to determine the value of the parameter $B_K(\mu)$ in the RI-MOM scheme, we studied the relevant matrix element on four lattices for which the main features are summarized in table 1. For technical details related to the production of the unquenched configurations, consult refs. [32, 33]. The values of the Wilson hopping parameters are listed in table 2, together with the corresponding masses of pseudoscalar mesons and ratios of the pseudoscalar to vector meson masses. In our notation the meson masses in lattice units are written in capital letters, whereas the small case letters are used to denote the same masses in physical units (e.g. $M_P = am_P$). As usual, the meson masses are extracted from the exponential fall-off of the two-point correlation functions. All the source operators used in this work are local. Statistical errors are estimated by using the jackknife procedure. For completeness, we give for each lattice the value of the critical hopping parameter as obtained by linearly extrapolating the square of the pseudoscalar meson mass (i.e. the valence quark mass) to zero. Using the same labels as in table 1, we give:

$$\kappa_{\rm crit} = \Big\{ \underset{\text{(a)}}{0.13580(2)},\ \underset{\text{(b1)}}{0.15931(6)},\ \underset{\text{(b2)}}{0.15885(7)},\ \underset{\text{(c)}}{0.15494(5)} \Big\}\,, \tag{2.1}$$

[5] As compared to ref. [26], our action is non-perturbatively improved whereas their was tree-level improved. In addition, we do not rotate the quark fields. This rotation is unnecessary (the 4-fermion operator is not improved) and it introduces a statistical noise. This is why our statistical errors are smaller than theirs.

Label	$(\beta,\ L^3 \times T)$	c_{SW}	Statistics	$a^{-1}(m_{K^*})$[GeV]	Comment
(a)	$(6.2,\ 24^3 \times 48)$	1.614	200 config.'s	2.71(8)	Quenched
(b1)	$(5.6,\ 24^3 \times 40)$	0	60 config.'s	2.48(11)	P.U. ($\kappa_{\rm sea} = 0.1575$)
(b2)	$(5.6,\ 24^3 \times 40)$	0	60 config.'s	2.60(12)	P.U. ($\kappa_{\rm sea} = 0.1580$)
(c)	$(6.1,\ 24^3 \times 40)$	0	60 config.'s	2.51(7)	Quenched

Table 1: Characteristics of the lattices used in this work. P.U. stands for the partial unquenching where the value of the corresponding dynamical (sea) quark mass parameter is indicated in the parentheses. a^{-1} has been estimated from the mass of the K^* meson in a way explained in ref. [34].

	κ_1	κ_2	κ_3	κ_4
(a)	0.1352	0.1349	0.1344	–
M_P	0.200(2)	0.245(2)	0.307(1)	–
m_P/m_V	0.592(23)	0.678(16)	0.758(9)	–
(b1)	0.1580	0.1575	0.1570	0.1560
M_P	0.230(5)	0.268(5)	0.303(4)	0.366(4)
m_P/m_V	0.612(30)	0.685(22)	0.735(17)	0.802(11)
(b2)	0.1580	0.1575	0.1570	0.1560
M_P	0.190(7)	0.234(5)	0.273(4)	0.342(3)
m_P/m_V	0.633(15)	0.700(16)	0.748(15)	0.816(11)
(c)	0.15363	0.15338	0.15311	0.15258
M_P	0.228(5)	0.248(4)	0.269(4)	0.307(4)
m_P/m_V	0.623(12)	0.662(11)	0.698(10)	0.752(8)

Table 2: Pseudoscalar meson masses (in lattice units), and the ratio of the pseudoscalar-to-vector mesons consisting of two degenerate (valence) quarks of mass corresponding to the indicated value of the hopping parameter κ_i. Labels (a,b1,b2,c) refer to the lattices with characteristics given in table 1.

The *physical* volume of the lattice (c) has been chosen to be nearly equal to the one used in simulations with partial unquenching ($L^3 \approx (1.9\,\mathrm{fm})^3$). Moreover, the hopping parameters for run (c) are chosen in such a way that the masses of pseudoscalar mesons are very close to the ones obtained from the unquenched runs. In this way any sizable difference between the physical quantities obtained from the quenched and unquenched simulations would signal the quenching error.

Let us now turn to the definition of the operator $Q(\mu)$ that appears in eq. (1.2). Since the Wilson term in the fermion action explicitly breaks the chiral symmetry, an extra mixing of the operator Q with other dimension-6 operators occurs. Consequently, in order to match the bare operator computed on the lattice to the

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continuum one, one has to subtract the effects of this mixing. In addition, one also needs the overall multiplicative renormalization constant. In a compact form, this procedure can be written as [35, 36]

$$Q(\mu) = Z\left(\mu, g_0^2\right)\left(Q_1 + \sum_{i\neq 1}\Delta_i(g_0^2)Q_i\right), \tag{2.2}$$

where Q_i on the r.h.s. denotes a bare lattice operator from the basis of 4-fermion operators, which we choose to be:

$$\begin{aligned} Q \equiv Q_1 &= \bar{s}\gamma_\mu d\,\bar{s}\gamma_\mu d + \bar{s}\gamma_\mu\gamma_5 d\,\bar{s}\gamma_\mu\gamma_5 d\,,\\ Q_2 &= \bar{s}\gamma_\mu d\,\bar{s}\gamma_\mu d - \bar{s}\gamma_\mu\gamma_5 d\,\bar{s}\gamma_\mu\gamma_5 d\,,\\ Q_3 &= \bar{s}d\,\bar{s}d - \bar{s}\gamma_5 d\,\bar{s}\gamma_5 d\,,\\ Q_4 &= \bar{s}d\,\bar{s}d + \bar{s}\gamma_5 d\,\bar{s}\gamma_5 d\,,\\ Q_5 &= \bar{s}\sigma_{\mu\nu} d\,\bar{s}\sigma_{\mu\nu} d\,. \end{aligned} \tag{2.3}$$

To compute the subtraction ($\Delta_i(g_0^2)$) and the renormalization ($Z(\mu, g_0^2)$) constants non-perturbatively, we use the technique proposed and elaborated in refs. [35, 36], and work in the RI-MOM scheme in the Landau gauge. In table 3, we list the non-perturbatively determined values for $Z(\mu, g_0^2)$ and $\Delta_i(g_0^2)$ for all four lattices. The actual computation is performed at two values of the renormalization scale, $\mu a = 0.7, 1.4$, and for each value of the quark mass. For the presentation convenience, we linearly extrapolate each constant to the chiral limit, $\kappa_q \to \kappa_{\rm crit}$. This extrapolation is very smooth. In addition, we evolve $Z(\mu a)$ to the scale $\mu a = 1$ by using the NLO anomalous dimension coefficient (see eq. (1.3)) and by taking the two-loop expression for $\alpha_s(\mu)$, normalized at $\alpha_s(3.41/a)$ by the value obtained from the average plaquette, as prescribed in refs. [37, 38].[6] At the scale $\mu = 1/a$ we make a comparison with the results of one-loop perturbation theory. We use the perturbative expressions of ref. [39], and combine them with the results of ref. [40]. In the basis (2.3) and in the RI-MOM scheme they read:

$$\begin{aligned} Z\left(\frac{1}{a}, g_0^2\right) &= 1 - \frac{g_0^2}{4\pi}\left(3.674 - 0.742c_{SW} - 0.388c_{SW}^2\right),\\ Z\left(\frac{1}{a}, g_0^2\right)\cdot\Delta_i(g_0^2) &= c_i\frac{g_0^2}{4\pi}\left(0.767 - 0.795c_{SW} + 0.272c_{SW}^2\right), \end{aligned} \tag{2.4}$$

where $c_2 = -11/12$, $c_3 = -1/6$, $c_4 = c_5 = 1/3$. In the numerical computations, we use the boosted coupling, $g_0^2 \to \tilde{g}_0^2 = 6/(\beta\langle P\rangle)$,[7] where the average plaquette values

[6] We stress that the values of $Z(\mu)$ which are computed at two values of the renormalization scale, are consistent with the NLO anomalous dimension (1.3). The difference that arise from the evolution from the two scales to $\mu = 1/a$ is smaller than the statistical error and it has been included in the errors quoted in table 3.

[7] In the improved case, we use $c_{SW} = 1$, to be consistent with the 1-loop order perturbation theory.

Label	*Method*	$Z(\mu = 1/a, g_0^2)$	$\Delta_2(g_0^2)$	$\Delta_3(g_0^2)$	$\Delta_4(g_0^2)$	$\Delta_5(g_0^2)$
(a)	NP	0.62(1)	−0.058(6)	−0.018(3)	0.016(2)	0.003(2)
	PT	0.68	−0.041	−0.008	0.015	0.015
(b1)	NP	0.51(2)	−0.198(16)	−0.030(8)	0.060(17)	0.019(11)
	PT	0.45	−0.231	−0.042	0.084	0.084
(b2)	NP	0.52(2)	−0.203(12)	−0.039(8)	0.066(18)	0.013(10)
	PT	0.45	−0.230	−0.042	0.083	0.083
(c)	NP	0.55(2)	−0.152(8)	−0.028(6)	0.047(10)	0.007(5)
	PT	0.52	−0.173	−0.032	0.063	0.063

Table 3: Renormalization and subtraction constants (eq. (2.2)), evaluated non-perturbatively in the (Landau) RI-MOM scheme, extrapolated (linearly in m_q) to the chiral limit and evolved to $\mu = 1/a$ by using the NLO evolution. Non-perturbative results are directly confronted to the perturbative values. The labels coincide with those used in table 1.

are computed in our simulations:

$$\langle P\rangle = \Big\{ \overset{(a)}{0.6136},\ \overset{(b1)}{0.5725},\ \overset{(b2)}{0.5735},\ \overset{(c)}{0.6050} \Big\} . \tag{2.5}$$

The labels above figures are the same as in table 1. From table 3 we see that the non-perturbatively computed subtraction constants Δ_i are not very far from their perturbatively estimated values (except for the case of $\Delta_5(g_0^2)$ whose perturbative value is generally much larger than the non-perturbative one). The value of the non-perturbatively computed renormalization constant differs from the perturbative one by $(10 \div 15)\%$.

2.1 Computation of the $B_K(\mu)$ parameter

The strategy to compute the bag parameter is well known [41] and we only briefly recall it. The asymptotic behavior of the two- and three-point correlation function is given by the following expressions:

$$\begin{aligned}
\mathcal{C}^{(2)}_{JJ}(\vec{p},;t) &= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \left\langle J(\vec{x},t) J^\dagger(0,0)\right\rangle \xrightarrow{t\gg 0} \frac{\mathcal{Z}_J}{2E_J} e^{-E_J t} , \\
\mathcal{C}^{(3)}(\vec{p},\vec{q};t_{P_1},t_{P_2};\mu) &= \sum_{\vec{x}\vec{y}} e^{i(\vec{p}\cdot\vec{y}-\vec{q}\cdot\vec{x})} \Big\langle P_5(\vec{x},t_{P_2}) Q(\vec{0},0;\mu) P_5^\dagger(\vec{y},t_{P_1})\Big\rangle \\
&\xrightarrow{-t_{P_1},t_{P_2}\gg 0} \frac{\sqrt{\mathcal{Z}_P}}{2E_P(\vec{q})} e^{-E_P(\vec{q})t_{P_1}} \left\langle \bar{P}(\vec{q})|Q(\mu)|P(\vec{p})\right\rangle \times \\
&\quad \times \frac{\sqrt{\mathcal{Z}_P}}{2E_P(\vec{p})} e^{-E_P(\vec{p})t_{P_2}} ,
\end{aligned} \tag{2.6}$$

where the operator $Q(\mu)$ is given by eq. (2.2), $\sqrt{\mathcal{Z}_P} \equiv \langle 0|P_5|P\rangle$ ($P_5 = i\bar{q}\gamma_5 q$), $\vec{p}$, $\vec{q}$ are momenta given to the interacting pseudoscalar mesons. In all our simulations, we fix $t_{P_2} = 12$, while t_{P_1} is free. The elimination of the exponentials in the three-point function is achieved by considering the ratio

$$R(\vec{p},\vec{q};\mu) = \frac{\mathcal{C}^{(3)}(\vec{p},\vec{q};t_{P_1},t_{P_2};\mu)}{Z_A^2\ \mathcal{C}^{(2)}_{PP}(\vec{p};t_{P_2})\ \mathcal{C}^{(2)}_{PP}(\vec{q};t_{P_1})}\,. \tag{2.7}$$

We also divided by the axial current renormalization constant which we computed non-perturbatively by using the method of ref. [42]:[8]

$$Z_A(g_0^2) = \Big\{ \overset{(a)}{0.80(1)},\ \overset{(b1)}{0.81(1)},\ \overset{(b2)}{0.80(1)},\ \overset{(c)}{0.82(1)} \Big\}\,. \tag{2.8}$$

After a careful inspection of the ratio (2.7) for several different values of $\vec{p}$ and $\vec{q}$ on the lattice (a), we select the following non-equivalent combinations:

$$\begin{aligned} \vec{p} &= (0,0,0) \qquad \text{and} \qquad \vec{q} = \{(0,0,0),\ (1,0,0)\}\,,\\ \vec{p} &= (1,0,0) \qquad \text{and} \qquad \vec{q} = \{(1,0,0),\ (0,1,0)\}\,, \end{aligned} \tag{2.9}$$

where each component is given in units of $(2\pi/L)$. For the lattices (b1), (b2) and (c), we consider only the first two combinations. This is because the temporal axis of these lattices is rather small ($T = 40$) to accommodate a clear signal when the larger momenta are given to the "kaons". As usual, whenever possible, we average over equivalent kinematical configurations.

Since the Wilson term in the fermion action breaks chiral symmetry, in the chiral expansion of the matrix element (1.2), one also allow for a free term and the one proportional to the quark mass (i.e. to m_K^2):

$$\langle \bar{K}^0(q)|Q(\mu)|K^0(p)\rangle = \alpha\ +\ \beta\ m_K^2 + \gamma\ (p\cdot q) + \cdots\,. \tag{2.10}$$

where the dots stand for higher order terms in expansion. The coefficients α and β in eq. (2.10) are expected to be consistent with zero if the chiral behaviour of the operator $Q(\mu)$ is correct.

To investigate this point we follow ref. [41] and rewrite eq. (2.10) to the first order, in the following form:

$$R(\vec{p},\vec{q};\mu) = \alpha + \beta\cdot X + \gamma\cdot Y\,, \tag{2.11}$$

[8]Recent works in which Z_A and other bilinear quark operators' renormalization constants are computed by using the method of ref. [42], can be found in refs. [43, 44, 45]. In refs. [46, 47], Z_A has also been computed by using the hadronic Ward identities. The results of the two methods are consistent.

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(a)

$\vec{p}\ ,\ \vec{q}$	$(0,0,0),(0,0,0)$		$(0,0,0),(1,0,0)$	
κ_i	$Y \equiv X$	$R(\vec{p},\vec{q})$	Y	$R(\vec{p},\vec{q})$
κ_1	0.068(7)	0.040(6)	0.111(11)	0.068(10)
κ_2	0.102(8)	0.068(8)	0.149(12)	0.102(10)
κ_3	0.166(10)	0.119(12)	0.217(13)	0.159(12)

$\vec{p}\ ,\ \vec{q}$	$(1,0,0),(0,1,0)$		$(1,0,0),(1,0,0)$	
κ_i	Y	$R(\vec{p},\vec{q})$	Y	$R(\vec{p},\vec{q})$
κ_1	0.182(17)	0.111(17)	0.066(7)	0.033(17)
κ_2	0.216(17)	0.150(18)	0.100(8)	0.063(16)
κ_3	0.283(17)	0.205(18)	0.163(10)	0.108(14)

Table 4: Numerical values of the quantity Y and of the ratio $R(\vec{p},\vec{q};\mu)$, computed in the RI-MOM scheme at $\mu a = 0.7$ ($\mu = 1.9\,\mathrm{GeV}$) for the lattice (a). The results are presented for each value of the hopping parameter κ_i specified in table 2, and for four combinations of momenta $\vec{p}$ and $\vec{q}$.

where the suitable choice for the fit with lattice data is

$$X \;=\; \frac{8}{3}\left|\frac{\sum_{\vec{x}}\langle A_0(x)P_5(0)\rangle}{\sum_{\vec{x}}\langle P_5(x)P_5(0)\rangle}\right|^2 \qquad \text{and} \qquad Y = \frac{p\cdot q}{m_K^2}\,X\,. \tag{2.12}$$

It is now clear that the division by Z_A^2 in the ratio (2.7) provides the renormalization of the axial current ($A_0 = \bar{q}\gamma_0\gamma_5 q$) in the quantity X, which allows then to identify the term proportional to Y as $f_K^2(p\cdot q)$, where f_K is the (renormalized) kaon decay constant defined from $\langle 0|A_\mu|K^0(p)\rangle = i f_K p_\mu$.

Before discussing the physical results, we give in table 4 and 5 the explicit values of the quantity Y and the ratio $R(\vec{p},\vec{q};\mu)$. We stress that the renormalization and subtraction constants used in this computation are those obtained for each value of κ_q separately. We have also computed $R(\vec{p},\vec{q};\mu)$ by using the renormalization and subtraction constants extrapolated to the chiral limit. Since the extrapolation of these constants to the chiral limit is very smooth,[9] the net effect on our results is negligibly small (less than 1%). In computing $(p\cdot q)$ we used the latticized energy-momentum relation

$$\sinh^2\left(\frac{E_P(\vec{p})}{2}\right) = \sinh^2\left(\frac{M_P}{2}\right) + \sin^2\left(\frac{\vec{p}}{2}\right), \tag{2.13}$$

which describes well our data [34].

[9]The corresponding plots can be obtained from the authors.

(b1)

$\vec{p}$, $\vec{q}$	$(0,0,0),(0,0,0)$		$(0,0,0),(1,0,0)$	
κ_i	$Y \equiv X$	$R(\vec{p},\vec{q})$	Y	$R(\vec{p},\vec{q})$
κ_1	0.078(3)	0.070(26)	0.117(6)	0.101(34)
κ_2	0.108(3)	0.091(27)	0.151(5)	0.120(34)
κ_3	0.113(3)	0.113(28)	0.183(5)	0.143(30)
κ_4	0.200(4)	0.162(27)	0.244(5)	0.189(33)

(b2)

$\vec{p}$, $\vec{q}$	$(0,0,0),(0,0,0)$		$(0,0,0),(1,0,0)$	
κ_i	$Y \equiv X$	$R(\vec{p},\vec{q})$	Y	$R(\vec{p},\vec{q})$
κ_1	0.060(5)	-0.002(22)	0.101(10)	0.013(40)
κ_2	0.090(5)	0.024(22)	0.134(8)	0.051(35)
κ_3	0.121(5)	0.051(24)	0.166(7)	0.085(34)
κ_4	0.183(4)	0.108(28)	0.229(5)	0.148(34)

(c)

$\vec{p}$, $\vec{q}$	$(0,0,0),(0,0,0)$		$(0,0,0),(1,0,0)$	
κ_i	$Y \equiv X$	$R(\vec{p},\vec{q})$	Y	$R(\vec{p},\vec{q})$
κ_1	0.083(14)	0.056(28)	0.125(16)	0.083(27)
κ_2	0.100(15)	0.069(26)	0.143(16)	0.097(26)
κ_3	0.118(16)	0.083(24)	0.163(17)	0.114(25)
κ_4	0.155(17)	0.113(21)	0.202(20)	0.148(24)

Table 5: The same as in table 4 but for lattices (b1), (b2) and (c) in which we consider two combinations of the momenta $\vec{p}$ and $\vec{q}$.

3. Physical results

Our central result is obtained from the fit of our lattice data (a), listed in table 4, to the form given in eq. (2.11). This fit is illustrated in figure 1 and the results are:

$$\begin{aligned}
\mu a = 0.7: \quad & \alpha = -0.017(10)\,; \qquad \beta = 0.10(10)\,; \\
& B_K^{\text{RI-MOM}}(\mu) = \gamma = 0.713(67)\,, \\
\mu a = 1.4: \quad & \alpha = -0.009(9)\,; \qquad \beta = 0.10(10)\,; \\
& B_K^{\text{RI-MOM}}(\mu) = \gamma = 0.706(65)\,.
\end{aligned} \tag{3.1}$$

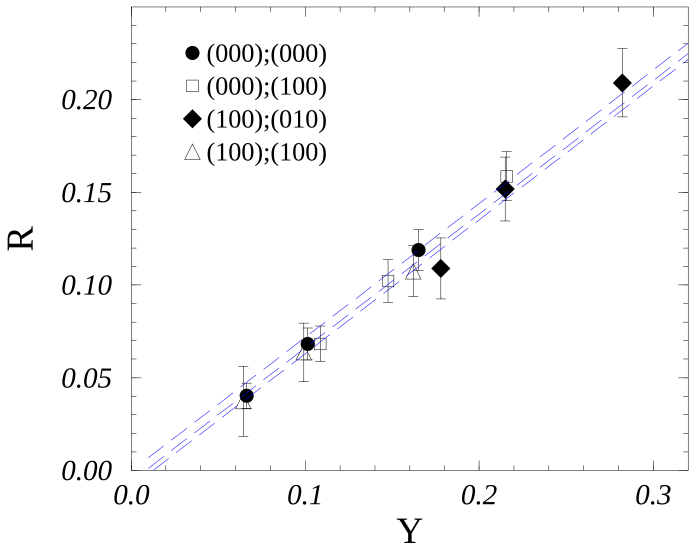

Figure 1: Fit of our lattice data (a) to eq. (2.11). The shown values of $R(\vec{p},\vec{q};\mu)$ are those corresponding to $\mu a = 0.7$, computed in the (Landau) RI-MOM scheme. The fitting curves (dashed lines) correspond to three different values of $X(\kappa_i)$ in eq. (2.11).

To convert $B_K^{\text{RI-MOM}}(\mu)$ to the RGI form and also to the $\overline{\text{MS}}$ (NDR) scheme, in which the lattice results are usually presented, we use eq. (1.3). At this point we should decide which coupling to use. One choice is to take the value $\alpha_s^{(n_f=0)}(1/a) = 0.178$ extracted from the average plaquette $\langle P\rangle$, by using the recipe of ref. [38]. Another (more phenomenological) choice is to relate the quenched $B_K^{\text{RI-MOM}}(\mu)$ to the physical (unquenched) world by using the physical $\alpha_s^{(n_f=4)}(1/a) = 0.266$, which is obtained by the NLO evolution of $\alpha_s(M_Z) = 0.118$. The difference between the two conversions is less than 1%. In this letter we use $\alpha_s^{(n_f=0)}(1/a)$, which leads to

$$B_K^{\overline{\text{MS}}}(2\,\text{GeV}) = 0.730(68)\,; \qquad \hat{B}_K = 1.009(94)\,. \tag{3.2}$$

As mentioned before, the results obtained at two different scales (3.1) are fully consistent with the first two coefficient of the anomalous dimension obtained in perturbation theory. The numbers given in eq. (3.2) are the average of the conversions $B_K^{\text{RI-MOM}}(\mu) \to B_K^{\overline{\text{MS}}}(2\,\text{GeV})$ from two different values of the scale μ. This is the main result of this letter to which we should add the systematic uncertainties which we now discuss.

- Our pseudoscalar mesons are composed of degenerated quarks. The quenched χPT suggests that the value of $B_K(\mu)$ may increase by $(3 \div 5)\%$ if we work with non-degenerate quarks [48]. To be on a safe side, we include +5% to the systematic uncertainty.

- We tried to modify the expansion (2.11), by including the term $X \log(m_P^2/f_P^2)$ which is the one that shows up in both the standard and the quenched χPT [49]. We also tried to modify (2.11) by adding a term proportional to X^2. Any of these two modifications does not change the value of $B_K(\mu)$.

- If instead of the procedure explained above, we use the renormalization and subtraction constants extrapolated to the chiral limit before doing the fits to eq. (2.11), the final value for $B_K(\mu)$ gets lower by less than a percent. To be conservative, we take $\pm 1\%$ which would also account for the tiny difference arising from the choice of $\alpha_s(\mu)$ in the conversion formulae.

- Since we work at fixed value of the lattice spacing it is difficult to estimate the discretization effects. Even though the action we use is free of $\mathcal{O}(a)$ effects, our operator $Q(\mu)$ is unimproved. To get some idea about the remaining $\mathcal{O}(a)$ uncertainty, we repeated the analysis by only improving the axial current, $A_0^{(\mathrm{impr.})} = A_0 + c_A \partial_0 P_5$ (with the value of $c_A = -0.04$ [31]). This further modifies the final $B_K(\mu)$ by $+1\%$.[10]

- Finally, one wonders about the size of the systematic uncertainty that comes from the use of quenched approximation. A comparison between the chiral expansion for the B_K parameter in the quenched and in the standard χPT, indicates that the effect of quenching might be negligible. In the next subsection, we will show that our unquenched data indeed tend to confirm that expectation.

By simply summing up all thee above (small) uncertainties, we end up with:

$$B_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.73(7)^{+0.05}_{-0.01}\,; \qquad \hat{B}_K = 1.01(9)^{+0.07}_{-0.01}\,, \tag{3.3}$$

which is our final result.

3.1 Unquenching

We now discuss the results of the analysis of our unquenched data. To our knowledge, this is the first attempt to compute the unquenched $B_K(\mu)$, by using Wilson fermions. In fact, our results are only partially unquenched (P.U.) which means that a valence quark mass is different from the dynamical one. Our both sets of data are obtained with $n_f = 2$ ((b1) and (b2) in table 1). As compared to the physical strange quark mass, the dynamical (sea) quark masses are, $m_{\mathrm{sea}}^{(b1)}/m_s \simeq 0.75$, and $m_{\mathrm{sea}}^{(b2)}/m_s \simeq 0.50$. The results of the fit to eq. (2.11) are given in table 6 and illustrated in figure (2).

[10] It can also be deduced from ref. [25] that the difference between B_K as obtained at $\beta \geq 6.1$ and the one extrapolated to the continuum limit is very small.

Label	μa	α	β	$\gamma = B_K^{\text{RI-MOM}}(\mu)$
(b1)	0.7	0.01(3)	0.06(19)	0.67(20)
	1.4	−0.01(3)	0.02(16)	0.65(17)
(b2)	0.7	−0.06(3)	0.28(24)	0.66(26)
	1.4	−0.08(3)	0.21(24)	0.65(26)
(c)	0.7	−0.01(3)	0.13(19)	0.69(18)
	1.4	−0.02(3)	0.09(19)	0.69(18)

Table 6: Result of the fit of our data obtained from the computation on the lattices (b1), (b2) and (c), to the form given in eq. (2.11).

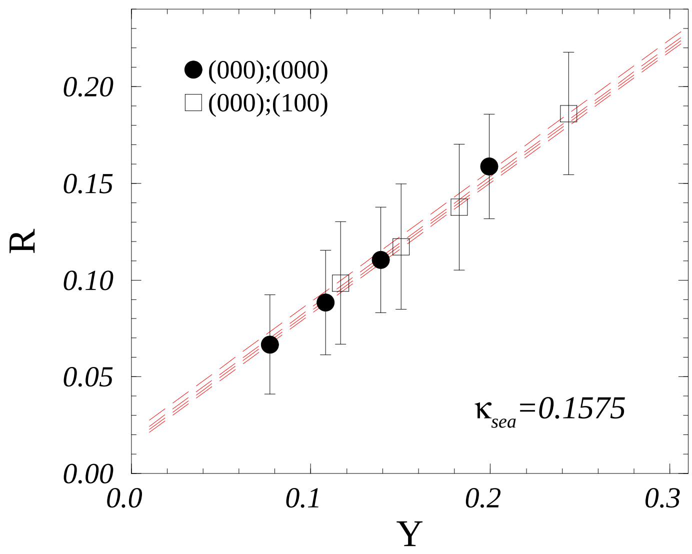

Figure 2: The same as for figure 1, but for the partially unquenched data (b1).

As in the previous subsection, we convert the obtained results to the RGI form and to the $\overline{\text{MS}}$ scheme. We use $\alpha_s^{(n_f=2)}(1/a) = 0.223$ and 0.222 for lattices (b1) and (b2) respectively, and $\alpha_s^{(n_f=0)}(1/a) = 0.187$ for the lattice (c), which as in the previous subsection are obtained from the average plaquettes. We finally have:

$$
\begin{aligned}
&(\text{b1}) \quad B_K^{\overline{\text{MS}}}(2\,\text{GeV}) = 0.67(19)\,; \qquad \hat{B}_K = 0.94(26)\,,\\
&(\text{b2}) \quad B_K^{\overline{\text{MS}}}(2\,\text{GeV}) = 0.67(26)\,; \qquad \hat{B}_K = 0.94(37)\,,\\
&(\text{c}) \quad B_K^{\overline{\text{MS}}}(2\,\text{GeV}) = 0.70(18)\,; \qquad \hat{B}_K = 0.97(25)\,.
\end{aligned} \tag{3.4}
$$

The statistical errors in these results are large. Systematics due to the lattice geometry is the same for both sets of data (quenched and unquenched), so that the difference would indicate the quenching errors.

First of all, from the above numbers ((b1) and (b2)) we do not observe any difference in B_K as the value of the sea quark mass changes. In principle, to get a fully unquenched result, one should work with more than two values of the dynamical quark mass, and then extrapolate B_K in such a way that the dynamical quark mass corresponds to the physical up/down quark. Since we do not see any change in $B_K(\mu)$ as the value of the sea quark mass change, we can not distinguish partially from the fully unquenched value for $B_K(\mu)$.

Even more, we do not see any significant effect due to the unquenching, i.e. by switching off the quark loops from the simulations (b1) and (b2), we get the results from the run (c), which totally agrees with the unquenched ones. This observation is on the same line with the conclusions drawn from the comparison of the standard and the (partially) quenched χPT [49, 50]. In ref. [51], however, by using the staggered fermions, an enhancement of B_K of the order of $(5\pm2)\%$ due to the unquenching has been reported. Obviously, with a poor statistical quality of our data, we are unable to confirm this result.[11]

In conclusion, we do not add any systematic uncertainty to our result (3.3) due to the use of quenched approximation.

4. Conclusion and prospects

In this letter, we computed the B_K parameter by using $\mathcal{O}(a)$ improved Wilson action but without improving the corresponding four fermion operator. From the non-perturbatively renormalized data, we obtained $B_K^{\overline{\rm MS}}(2\,\mathrm{GeV}) = 0.73(7)^{+0.05}_{-0.01}$, which confirms the tendency of larger values for this quantity when working with the Wilson fermions [25, 26, 39, 52]. In order to examine the effect of quenching, we computed B_K with dynamical fermions ($n_f = 2$). Even though the statistical quality of our unquenched results is poor, we do not see any significant deviation of the parameter B_K as compared to the one obtained in the quenched simulation. To quantify the small quenching errors (if any!) it is obviously important to increase the statistics of the unquenched data.

The main systematic uncertainty, which we could not estimate, is expected to come from the computation of the bare operator, i.e. from the subtractions of the effects of mixing with operators of the same dimension (peculiarity of the Wilson lattice regularization). This can be completely avoided if one follows the proposal of ref. [53], namely by using the Ward identity to relate $Q(\mu) = VV + AA$ to the parity violating operator $\mathcal{Q}(\mu) = VA + AV$ for which no subtraction is needed.

[11] In fact, the enhancement in B_K due to the unquenching $\sim 5\%$, as claimed by the authors of ref. [51], comes from their simulations with $n_f = 4$, while their data with $n_f = 2$ almost coincide with the quenched ones.

Alternatively, one can use the twisted mass fermions, as proposed in ref. [54]. It will be very interesting to see if the results for B_K without subtractions will confirm the values obtained in this letter.

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References

[1] D.E. Groom et al., *Review of particle physics*, *Eur. Phys. J.* **C 15** (2000) 1.

[2] A.J. Buras, *Weak hamiltonian, CP-violation and rare decays*, hep-ph/9806471.

[3] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [hep-ph/9512380].

[4] L. Wolfenstein, *Parametrization of the Kobayashi-Maskawa matrix*, *Phys. Rev. Lett.* **51** (1983) 1945.

[5] A.J. Buras, M.E. Lautenbacher and G. Ostermaier, *Waiting for the top quark mass, $K^+ \to \pi^+\nu\bar{\nu}, B_s^0 - \bar{B}_s^0$ mixing and CP asymmetries in B-decays*, *Phys. Rev.* **D 50** (1994) 3433 [hep-ph/9403384].

[6] M.K. Gaillard and B.W. Lee, *$\Delta I = 1/2$ rule for nonleptonic decays in asymptotically free field theories*, *Phys. Rev. Lett.* **33** (1974) 108.

[7] M. Ciuchini et al., *Next-to-leading order QCD corrections to $\Delta f = 2$ effective hamiltonians*, *Nucl. Phys.* **B 523** (1998) 501 [hep-ph/9711402].

[8] A.J. Buras, M. Misiak and J. Urban, *Two-loop QCD anomalous dimensions of flavour-changing four-quark operators within and beyond the standard model*, *Nucl. Phys.* **B 586** (2000) 397 [hep-ph/0005183].

[9] G. Altarelli, G. Curci, G. Martinelli and S. Petrarca, *QCD nonleading corrections to weak decays as an application of regularization by dimensional reduction*, *Nucl. Phys.* **B 187** (1981) 461.

[10] A.J. Buras, M. Jamin and P.H. Weisz, *Leading and next-to-leading QCD corrections to epsilon parameter and $B^0\bar{B}^0$ mixing in the presence of a heavy top quark*, *Nucl. Phys.* **B 347** (1990) 491.

[11] J.F. Donoghue, E. Golowich and B.R. Holstein, *The $\Delta s = 2$ matrix element for $K^0 - \overline{K^0}$ mixing*, *Phys. Lett.* **B 119** (1982) 412.

[12] W.A. Bardeen, A.J. Buras and J.M. Gerard, *The b parameter beyond the leading order of* $1/N$ *expansion*, *Phys. Lett.* **B 211** (1988) 343.

[13] S. Peris and E. de Rafael, $K^0 - \overline{K^0}$ *mixing in the* $1/N_c$ *expansion*, *Phys. Lett.* **B 490** (2000) 213 [hep-ph/0006146].

[14] J. Bijnens and J. Prades, *Scheme dependence of weak matrix elements in the* $1/N_c$ *expansion*, *J. High Energy Phys.* **01** (2000) 002 [hep-ph/9911392].

[15] S. Bertolini, J.O. Eeg, M. Fabbrichesi and E.I. Lashin, *The* $\Delta I = 1/2$ *rule and* $\hat{B}_K$ *at* $O(p^4)$ *in the chiral expansion*, *Nucl. Phys.* **B 514** (1998) 63 [hep-ph/9705244].

[16] N. Bilic, C.A. Dominguez and B. Guberina, *QCD calculation of* $K^0 - \overline{K^0}$ *mixing from three point function sum rules*, *Z. Physik* **C 39** (1988) 351.

[17] L.J. Reinders and S. Yazaki, *A QCD sum rule calculation of the* $K^0 - \overline{K^0}$ *mixing parameter b*, *Nucl. Phys.* **B 288** (1987) 789.

[18] J. Prades, C.A. Dominguez, J.A. Penarrocha, A. Pich and E. de Rafael, *The* $K^0 - \overline{K^0}$ *B-factor in the QCD hadronic duality approach*, *Z. Physik* **C 51** (1991) 287.

[19] M. Ciuchini, G. D'Agostini, E. Franco, V. Lubicz, G. Martinelli, F. Parodi, *2000 CKM-triangle analysis: a critical review with updated experimental inputs and theoretical parameters*, hep-ph/0012308.

[20] S.R. Sharpe, *Chiral perturbation theory and weak matrix elements*, *Nucl. Phys.* **53** *(Proc. Suppl.)* (1997) 181 [hep-lat/9609029].

[21] L. Lellouch, *Light hadron weak matrix elements*, hep-lat/0011088.

[22] V. Lubicz, *Phenomenology of the standard model from lattice QCD*, hep-ph/0010171.

[23] JLQCD collaboration, *Kaon B parameter from quenched lattice QCD*, *Phys. Rev. Lett.* **80** (1998) 5271 [hep-lat/9710073].

[24] G. Kilcup, R. Gupta and S.R. Sharpe, *Staggered fermion matrix elements using smeared operators*, *Phys. Rev.* **D 57** (1998) 1654 [hep-lat/9707006].

[25] JLQCD collaboration, *The kaon B-parameter with the Wilson quark action using chiral ward identities*, *Phys. Rev.* **D 60** (1999) 034511 [hep-lat/9901018].

[26] L. Conti, A. Donini, V. Gimenez, G. Martinelli, M. Talevi and A. Vladikas, *Lattice B-parameters for* $\Delta s = 2$ *and* $\Delta I = 3/2$ *operators*, *Phys. Lett.* **B 421** (1998) 273 [hep-lat/9711053].

[27] RBC collaboration, $K \to \pi\pi$ *decays with domain wall fermions: lattice matrix elements*, hep-lat/0011042.

[28] CP-PACS collaboration, *Kaon B-parameter from quenched domain-wall QCD*, hep-lat/0010079.

[29] RBC collaboration, *Non-perturbative renormalisation with domain wall fermions*, hep-lat/0011036.

[30] Y. Zhestkov, *Domain wall fermion study of scaling in non-perturbative renormalization of quark composite operators*, hep-lat/0011002.

[31] M. Luscher, S. Sint, R. Sommer, P. Weisz and U. Wolff, *Non-perturbative $O(a)$ improvement of lattice QCD*, *Nucl. Phys.* **B 491** (1997) 323 [hep-lat/9609035].

[32] SESAM collaboration, *Critical dynamics of the hybrid Monte Carlo algorithm*, *Nucl. Phys.* **63** *(Proc. Suppl.)* (1998) 946 [hep-lat/9712020].

[33] SESAM collaboration, *Light and strange hadron spectroscopy with dynamical Wilson fermions*, *Phys. Rev.* **D 59** (1999) 014509 [hep-lat/9806027].

[34] D. Becirevic et al., *Light hadron spectroscopy on the lattice with the non-perturbatively improved Wilson action*, hep-lat/9809129.

[35] M. Crisafulli et al., *Chiral behaviour of the lattice b_k-parameter with the wilson and clover actions at $\beta = 6.0$*, *Phys. Lett.* **B 369** (1996) 325 [hep-lat/9509029].

[36] A. Donini, V. Gimenez, G. Martinelli, M. Talevi and A. Vladikas, *Non-perturbative renormalization of lattice four-fermion operators without power subtractions*, *Eur. Phys. J.* **C 10** (1999) 121 [hep-lat/9902030].

[37] G.P. Lepage and P.B. Mackenzie, *On the viability of lattice perturbation theory*, *Phys. Rev.* **D 48** (1993) 2250 [hep-lat/9209022].

[38] C.T.H. Davies, K. Hornbostel, G.P. Lepage, P. McCallum, J. Shigemitsu and J.H. Sloan, *Further precise determinations of α_s from lattice QCD*, *Phys. Rev.* **D 56** (1997) 2755 [hep-lat/9703010].

[39] R. Gupta, T. Bhattacharya and S. Sharpe, *Matrix elements of 4-fermion operators with quenched Wilson fermions*, *Phys. Rev.* **D 55** (1997) 4036 [hep-lat/9611023].

[40] S. Capitani et al., *Local bilinear operators on the lattice and their perturbative renormalisation including $O(a)$ effects*, hep-lat/9711007.

[41] M.B. Gavela et al., *The kaon B parameter and $K\pi$ and $K\pi\pi$ transition amplitudes on the lattice*, *Nucl. Phys.* **B 306** (1988) 677.

[42] G. Martinelli, C. Pittori, C.T. Sachrajda, M. Testa and A. Vladikas, *A general method for nonperturbative renormalization of lattice operators*, *Nucl. Phys.* **B 445** (1995) 81 [hep-lat/9411010].

[43] M. Gockeler et al., *Nonperturbative renormalisation of composite operators in lattice QCD*, *Nucl. Phys.* **B 544** (1999) 699 [hep-lat/9807044].

[44] V. Gimenez, L. Giusti, F. Rapuano and M. Talevi, *Non-perturbative renormalization of quark bilinears*, *Nucl. Phys.* **B 531** (1998) 429 [hep-lat/9806006].

[45] D. Becirevic, V. Lubicz, G. Martinelli and M. Testa, *Quark masses and renormalization constants from quark propagator and 3-point functions*, *Nucl. Phys.* **83** *(Proc. Suppl.)* (2000) 863 [`hep-lat/9909039`].

[46] M. Luscher, S. Sint, R. Sommer and H. Wittig, *Non-perturbative determination of the axial current normalization constant in $O(a)$ improved lattice QCD*, *Nucl. Phys.* **B 491** (1997) 344 [`hep-lat/9611015`].

[47] T. Bhattacharya, R. Gupta, W. Lee and S. Sharpe, *Order a improved renormalization constants*, `hep-lat/0009038`.

[48] S.R. Sharpe, *Phenomenology from the lattice*, `hep-ph/9412243`.

[49] S.R. Sharpe, *Quenched chiral logarithms*, *Phys. Rev.* **D 46** (1992) 3146 [`hep-lat/9205020`].

[50] M.F. L. Golterman and K.-C. Leung, *Applications of partially quenched chiral perturbation theory*, *Phys. Rev.* **D 57** (1998) 5703 [`hep-lat/9711033`].

[51] G. Kilcup, D. Pekurovsky and L. Venkataraman, *On the N_f and a dependence of B_K*, *Nucl. Phys.* **53** *(Proc. Suppl.)* (1997) 345 [`hep-lat/9609006`].

[52] (UKQCD) collaboration, L. Lellouch and C.J.D. Lin, $\Delta s = 2$ *and* $DeltaI = 3/2$ *matrix elements in quenched QCD*, *Nucl. Phys.* **73** *(Proc. Suppl.)* (1999) 312 [`hep-lat/9809142`].

[53] D. Becirevic et al., $K^0 - \overline{K^0}$ *mixing with Wilson fermions without subtractions*, *Phys. Lett.* **B 487** (2000) 74 [`hep-lat/0005013`].

[54] R. Frezzotti, P.A. Grassi, S. Sint and P. Weisz, *A local formulation of lattice QCD without unphysical fermion zero modes*, *Nucl. Phys.* **83** *(Proc. Suppl.)* (2000) 941 [`hep-lat/9909003`].